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# FRACTURE OF ROCKS IN SOLID MEDIUM:—A CONSIDERATION OF THE OCCURRENCE OF THE EARTHQUAKE SEQUENCES

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**Abstract**

The fracture phenomena of rocks in the solid mediums was studied. A large quantity of residual stress was observed just after the major fracture. The ratio of stress drop to initial stress was large for the weak mediums and small for the strong mediums. The effect of existence of free surface on the strength of specimen, the stress drop ratio at the moment of fracture, and the stress relaxation after the major fracture was also quite remarkable.

The three features of the earthquake occurrence was mentioned. (1) Aftershock sequences occur in accordance with Omori's formula. (2) Earthquakes occur generally in groups. (3) The earthquake group changes its type with focal depth from aftershock type to earthquake-cluster type, and finally to isolatedshock type. The above features were well explained with the results of the rock fracture in the solid mediums.

## 1. Introduction

It has been the quite difficult problem why the earthquakes occur. This problem includes the two questions, that is, the origin of the earthquake-generating stress and the stress system acting in the seismic region. About the mechanism of the earthquake occurrence, that is, about the question how the earthquakes occur, there have been many hypotheses.

Among these hypotheses, the elastic rebound theory has been the most famous one explaining the crustal deformation and the genesis of the seismic faulting. Knopoff [1966] and Kasahara [1966] proposed independently the simulating models for the seismic activity. These models are made up of the rheological elements, so that it is easy to explain the earthquake phenomena quantitatively with the numerical calculation. Similar kind of models have also been proposed by many authors (e.g., Matsuzawa [1952]). Griggs [1960] and Orowan [1960] suggested the hypothesis of shear melting slip for the fault mechanism of the intermediate and deep focus earthquakes. Mogi [1962] classified the shallow focus earthquakes in three types, that is, fore-main-aftershock type, main-aftershock type, and earthquake-swarm type. He concluded from his experimental study that the occurrence of such three types closely relates to the inhomogenuity of the medium of the seismic region.

The models and hypotheses for the earthquake phenomena must interpret consistently the laws and features of the earthquake occurrence. Omori's law of aftershock sequence is one of the most famous one. Kusakabe [1927] and recently Benioff [1951] interpreted this law with the recovering behavior of the creep of rocks at the fault region. This idea essentially attributes the aftershock phenomena to the visco-elastic behavior of the seismic region.

In this report, the laws and features of the earthquake occurrence will first be mentioned in section 2. In section 3, the study of the experimental demonstration of the fracture phenomena of rocks in the solid mediums will be stated.

## 2. Classification of the Earthquake Group

Orowan [1960] pointed out that the earthquakes occur in general in groups and this fact is the remarkable feature of the earthquakes differing from the other kind of fracture phenomena. This may be adopted as a law of earthquake occurrence. The aftershock sequence is one type of such earthquake groups. As mentioned above, Omori's law of aftershock sequence is another remarkable law of earthquake occurrence.

However, the earthquake groups do not always occur in such typical way. It has been observed that many earthquakes occurred in groups irregularly with the lapse of time and about the magnitude. It is widely accepted that the type of aftershock sequence occurs at rather shallow zone of the seismic region. This nature will be analized more clearly in the following.

It seems to be better that the earthquake groups are classified in the following four types. (1) Type of aftershock sequence : Occasionally main shock is preceded by the insignificant minor foreshocks. (2) Type of earthquake-cluster : A few or more earthquakes having similar magnitude occur at the same region successively but irregularly. (3) Type of earthquake-swarm : Innumerable but in general small earthquakes occur one after another at relatively limited area. (4) Type of isolatedshock : Earthquakes seem to be quite isolated spatially and in curse of time, and independent of one another at all. Of course, in general, this isolation might be merely imaginary because it depends strongly on the sensitivity of the seismograph. However, for the intermediate and deep focus earthquakes, this solitariness seems to be real nature. This problem will be dicussed later again.

For the statistical treatment, 115 major earthquake groups, which occurred in

1959 and 1960 in and near Japan, were kaken up from the Seismological Bulletin of the Japan Meteorological Agency. These groups were selected according to the restriction condition that the maximum felt distance of the earthquake group exceeds 150 km. These earthquake groups were clssified in four types and then each of four was divided into the intervals of focal depth of every 20 km. The focal depth of shock origin is not so much accurate, that the present method for the division of depth will be too much fine and the result of analysis is not so reliable. However, this method was convenient to know the tendency which type of earthquake group is predominant at some depth. The results are listed in Table 1. The epicenters of 115 major earthquake groups are shown on geographycal map in Fig. 1. The area was subdivided into two parts, A and B. The area A is the Pacific coast of North-Eastern Japan and its vicinity, and the area B is the rest

Table 1. Classification of the earthquake groups in types and focal depths

| Type / Depth | | Type of Isolated-shock | Type of Earthquake-cluster | Type of Aftershock-sequence | Type of Earthquake-swarm | Total |
|---|---|---|---|---|---|---|
| 0-20km | A | 3 | 1 | 5 | 0 | |
| | B | 3 | 2 | 8 | 2 | |
| | total | 6 | 3 | 13 | 2 | 24 |
| 20<-40 | A | 9 | 13 | 4 | | |
| | B | 2 | 0 | 0 | | |
| | total | 11 | 13 | 4 | ? | 28 |
| 40<-60 | A | 6 | 10 | | | |
| | B | 3 | 0 | | | |
| | total | 9 | 19 | 0 | ? | 28 |
| 60<-80 | A | 7 | 8 | | | |
| | B | 1 | 0 | | | |
| | total | 8 | 8 | 0 | ? | 16 |
| 80<-120 | A | 7 | 3 | 1 | | |
| | B | 2 | 0 | 0 | | |
| | total | 9 | 3 | 1 | ? | 13 |
| 120< | A | 1 | | | | |
| | B | 5 | | | | |
| | total | 6 | 0 | 0 | ? | 6 |
| Total | | 49 | 46 | 18 | 2 | 115 |

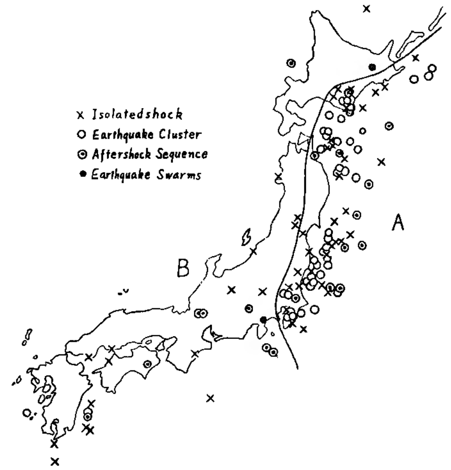


Fig. 1. The major earthquake groups which occurred in and near Japan during 1959-1960.

area. The tendency of the earthquake occurrence between these two areas has been thought to be quite different.

Katsumata's formula (Katsumata [1963]) of the minimum magnitude of the observable shock at any seismological station and with any instrument indicates that the effect of the focal depth on the detection of minor shock is quite negligible for the depths of 0–100 km and the epicentral distances of 100–500 km.

From the Table 1, it is easily seen that the type of aftershock sequence is quite concentrated at the depth of 0–20 km independently of the area A or B. It seems that the type of earthquake-swarm also occurs at the shallowest zone, though only two samples are listed. The type of earthquake-cluster, on the other hand, are mainly distributed at the depths of 20-80 km and have maximum at the zone of 40-60 km. At first glance, the isolatedshocks seem to be distributed independently of the focal depth. However, in the seismic zone deeper than about 80 km, the ratio of the isolated-shocks to the total earthquake groups becomes almost unity, while at the shallower zones this ratio is much small. By the recent precise observations of the microearthquakes, even much smaller shocks than the present ones were accompanied with the foregoing or the following shocks (1964 [Watanabe]). Then it might be safe to say that the tendency of isolation of shocks is the real nature of the intermediate and deep focus earthquakes.

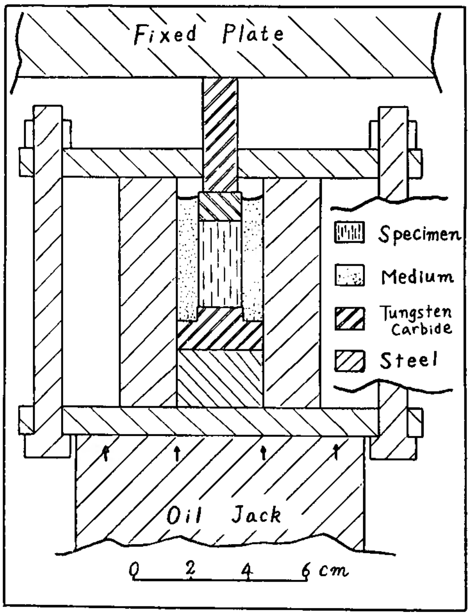


Fig. 2. Testing apparatus for the study of the fracture of rocks in the solid mediums.

It is concluded that from the surface to the deeper region of the earth, the feature of the earthquake occurrence is considerably changed. At the zone 0–20 km, the aftershock type is predominant. The ratio of this type to total decreases rapidly with depth, and the type of earthquake-cluster becomes predominantly at the zone 40–60 km. At the zone deeper than 80 km, the grouping nature of shocks is almost vanished and each earthquake occurs independently of one another.

## 3. Fracture of Rocks in the Solid Mediums

From the viewpoint of the fracture phenomena of materials, above mentioned features of the earthquake occurrence are stated in other wards as follows, supposing that the earthquake-generating stress is accumulated gradually and the shocks occur at the region of high stress concentration. (1) Stress is not completely dropped at the moment of a major rupture and a large quantity of stress is remained after that time, so that the successive ruptures can be followed making the aftershock sequence or the earthquakecluster. (2) This remained stress just after the major rupture (which will be called 'remained-stress' hereafter) is gradually (and exponentially) released. (3) It seems that the ratio of stress drop to applied initial stress becomes rapidly small with depth of the fault region, and the tendency of the exponential stress-relaxation, which will be connected with the aftershock sequence, is also diminished.

In this section, it is demonstrated experimantally that the above nature of the fracture phenomena will be explained by considering the fracture of rocks in the solid (visco-elastic) mediums.

Fig. 2 shows the testing apparatus for the present study. Cylindrical specimens of granite were set at the center of the steel cylinder. The size of the cylinder chamber was 30 mm in inner diameter and 60 mm height. The space between the specimen and the cylinder wall was filled with various solid mediums.

Table 2. Discription of the solid mediums

| Medium | Remarks | Compressive Strength | Rheological Property |
|---|---|---|---|
| Pitch | | ......... | Viscous body |
| Turpentine | Nakarai Chemicals Japan | 40kg/cm$^2$ | Maxwell type Visco-elastic body |
| Lakecide Cement NO 70 C | Hugh Courtright Co., U. S. A. | 250 | Maxwell type Visco-elastic body |
| Polyester Resin | | 880 (Yield Strength) | Plastic body |
| Pyrophyllite | American Lave Corp., U. S. A. | 365 | Voigt type Visco-elastic body |
| Marble (fine grain) | Mine, Yamaguchi, Japan | 1100 | Voigt type Visco-elastic body |

Table 3. Results of rock fracture in the various solid mediums

| No. | Medium | Remarks | Strength of Rock Specimen | Stress Drop | Ratio of Stress Drop to Initial Stress | Residual stress* |
|---|---|---|---|---|---|---|
| 3 | Pitch | Standard test | 1,100 kg/cm$^2$ | 980 kg/cm$^2$ | 0.89 | 0 kg/cm$^2$ |
| 18** | Turpentine | Standard test | 4,900 | 2,100 | 0.43 | 2,800 |
| 5 | 〃 | Medium was sticked to cylinder wall | 8,400 | 7,500 | 0.89 | 0 |
| 4 | Lakecide Cement 70C | Standard test | 4,900 | 270 | 0.055 | 0 |
| 15 | 〃 | 10 mm diameter specimen | 900 | 250 | 0.28 | 0 |
| 16 | 〃 | 20 mm diameter specimen | 5,500 | (30) | (0.005) | 1,400 |
| 10 | Polyester Resin | Standard test | 9,000 (Yield strength) | 1,300 | 0.08 | 11,000 |
| 7 | 〃 | Large gap between medium and wall | 1,300 | 980 | 0.75 | 0 |
| 9 | Pyrophyllite | Standard test | 8,400 | 3,500 | 0.41 | 2,200 |
| 11 | Marble | Standard test | 18,000 | (400) | (0.02) | ……… |
| 12 | 〃 | Lubricators were used | 11,500 | 9,500 | 0.83 | 1,800 |

* Residual stress means the stress which was remained after the lapse of enough time.

** This run only carried out at much lower temperature than the others.

Axial compressive stress was applied to the specimen with the tungsten carbide plunger which was driven by the oil jack. Deformation of the specimen (piston displacement) was measured with dial gage. Upper part of the cylinder was left open, then the medium could be escaped only to this direction. By the thrust of the plunger, specimen is shortened to the axial direction and is expanded to the lateral direction. However, this lateral displacement will be suppressed by the filling solid medium, so that the fracture of specimen becomes difficult. The medium in turn will be oppressed and deformed elastically at first and then flow visco-elastically.

Of course, above stated behavior entirely depends on the rheological property of the medium. Pitch, turpentine, solid balsam (Lakecide cement No70C), polyester resin, pyrophyllite and marble were used as the filling mediums. Details of the mediums are listed in Table 2. The experimental results are summarized in Table 3. To know the effect of mechanical properties of medium on the fracture phenomena, the 'standard test' was established. Its conditions were as follows: (1) The size of specimen is 15 mm in diameter and 30 mm long. (2) The medium is not sticked with the cylinder wall, but there must not be even slight clearance between medium and the wall. (3) No lubricating materials are used at any contacting surface.

From the results of the standard tests, it was concluded that the higher the strength of the medium was, the higher the rupture strength of the specimen became. Stress drop at the moment of rupture was also closely related with the mechanical property of the surrounding medium. The stronger the medium was, the more quantity of stress was remained. The relations of the ram pressure (applied stress) and the piston-displacement for the specimens in the various mediums are shown in Fig. 3.

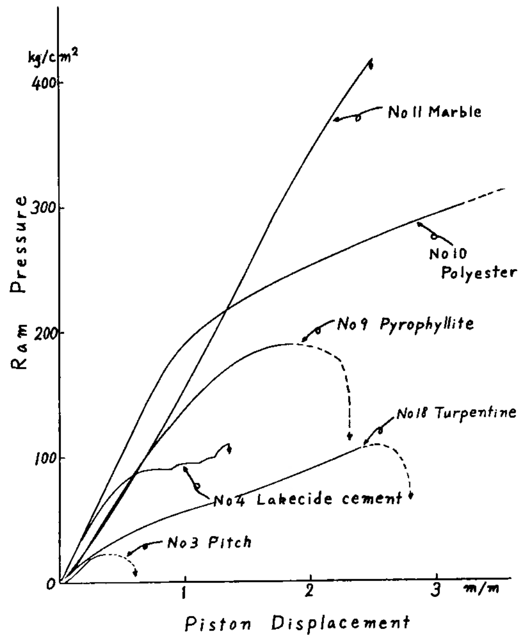


Fig. 3. Stress (ram pressure)-deformation (piston displacement) relations for the standard tests in the various mediums.

For the viscous or the Maxwell-visco-elastic mediums, it was observed that the 'remained stress' just after the main rupture was released gradually with the lapse of time, and a quite small quantity of residual stress was remained after the enough time. On the other side, for the Voigt-visco-elastic or the plastic mediums, the 'remained stress' was hardly relaxed and a lot of residual stress was remained. The piston displacements of the process of the stress relaxation are shown in Fig. 4 a) for the specimens in the various mediums, and both the deformation and the relating stress-relaxation in Lakecide cement are shown in Fig. 4 b). For many cases, it was observed that the specimens did not deform continuously in course of time, but discontinuously. Two remarkable examples are shown in Fig. 5 a) and b). The

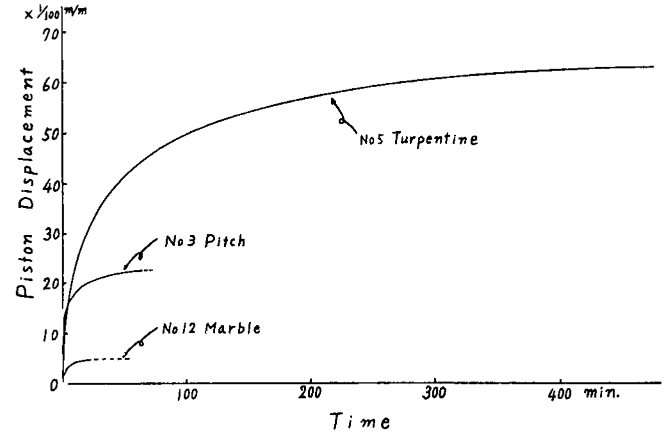


Fig. 4 a). Time-deformation relations for the specimens in three mediums after the major fracture.

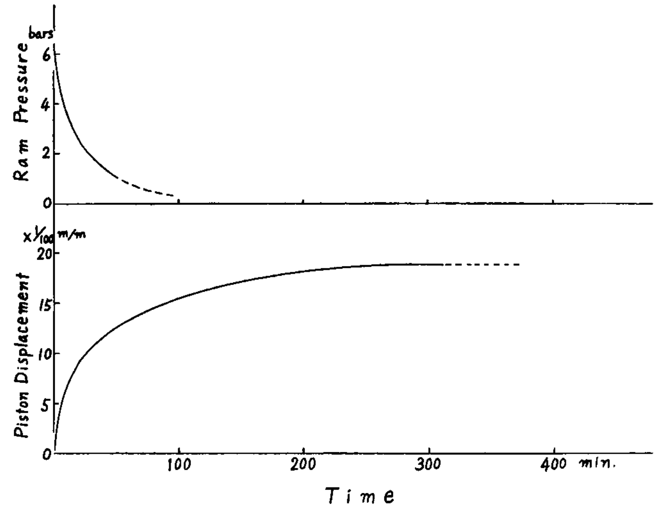


Fig. 4 b). Stress relaxation and relating deformation for the specimen in Lakecide cement 70C after the major fracture (Run No. 15).

number-time relations for such discontinuous processes of deformation or stress relaxation are quite similar to that of aftershock sequences. The result corresponding to Fig. 5 a) is shown in Fig. 6.

The other important factors for the fracture phenomena are the existence of the freely movable surface and the solid friction. If there is wide free surface near the highly stressed region, the fracture may be easily occurred at that stressed field. To find out the effect of the existence of the free surface, the tests in which the size of specimen was varied were carried out. The granite specimens, 10 mm in diameter 20 mm long, 15 mm in diameter 30 mm long, and 20 mm in diameter 40 mm long were tested in solid balsam (Lakecide cement 70C). The results are also listed in Table 3. As was expected beforehand

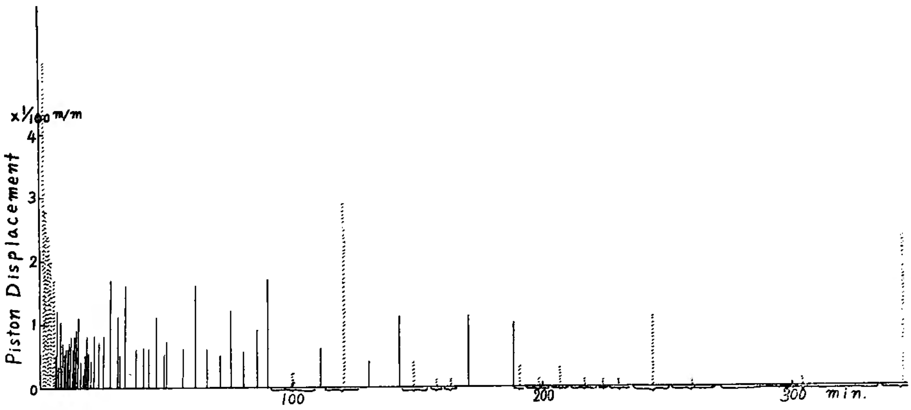


Fig. 5 a)

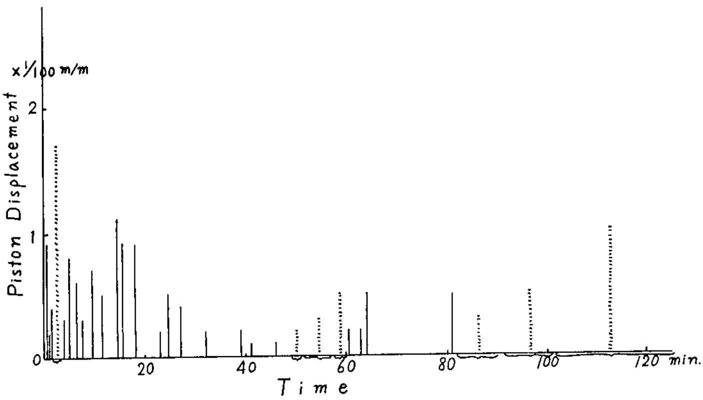


Fig. 5 b)

Fig. 5 a). Discontinuous piston displacements which followed successively after the major fracture of specimen in turpentine (Run No. 5), and Fig. 5 b) in Lakecide cement 70C (Run No. 15).

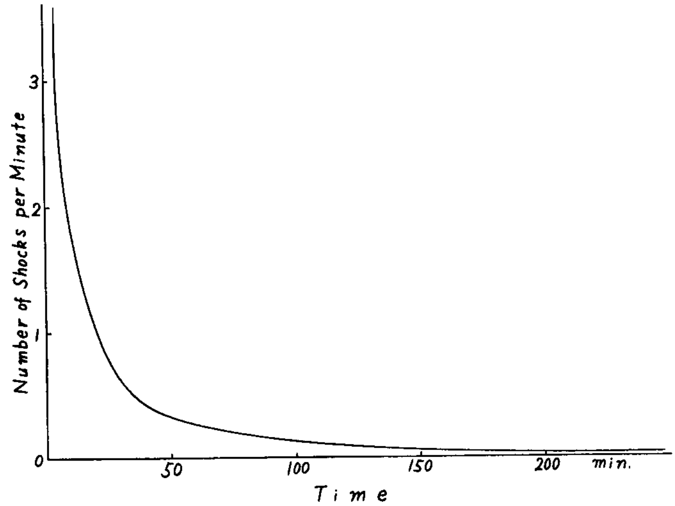


Fig. 6. Relation of time and shock number per unit time for the stress relaxation (Run No. 15).

it is seen from the table that the effect of the size of the specimen is quite remarkable. If the highly stressed region is firmly surrounded and the escape of the medium is difficult, the strength of that region will be extremely increased and the stress drop ratio at the moment of rupture will be decreased.

Another test for the demonstration of the effect of free surface was done. In this test, sufficient clearance was made between the medium and the cylinder wall. Polyester resin was used as the filling medium. The result is shown in Table 3. In this case, the effect of free surface is also quite remarkable. It will be concluded that the more wide the free surface is, the lower the strength of the specimen becomes.

To test the effect of solid friction, the lubricating materials, that is, lead foil and molibdenum disulphide grease, were used between the cylinder wall and marble which was used as the filling medium. By the effect of these lubricants, the strength of the specimen was considerably decreased and the stress drop ratio became large. The result is also listed in Table 3,

## 4. Discussion

(*1*) Since the strength of medium at the seismic region is very important, it will be discussed at first. There are four important factors for the strength of the seismic region, that is, earth's crust and upper mantle. These factors are pressure, temperature, constituent rocks, and the existence of vapor and other fluids.

Effects of pressure and temperature on the strength of rocks have been studied by many authors (e. g. Robertson [1955], Griggs et al. [1960], Matsushima

[1960, 1961]). The strength of sillicate rocks increases rapidly with depth by the effect of increasing pressure, reaches the maximum at the depth of 30-40km, and then decreases gradually by the effect of increasing temperature. The type of the constituent rocks varies considerably from the surface of the earth's crust to the upper mantle. These are low consolidated rocks, consolidated rocks, granitic rocks, gabbro and its group, and ultramafic rocks. According to this order from the surface to the interior, the strength of these rocks also becomes higher. The effect of water content on the strength of rocks must not be neglected. The existence of vapor or other fluids decreases the solid friction and lowers the rupture strength of rocks under the pressure. It is expected that there is plenty of water in the superficial layers of the earth. The quantity of water content will be lessened rapidly with depth by the effect of increasing temperature, so that the strength of rocks near the earth's surface is much lower than that at the deep part of the earth's lithosphere.

Summarizing the above effects, the strength of the earth's crust and upper mantle at first increases extremely rapidly with depth, reaches the maximum at the depth of 30-40 km, and then gradually decreases.

(*2*) Since the effect of free surface on the fracture phenomena is diminished at the deep part of the seismic region and the strength of the surrounding medium becomes extremely high as was mentioned above, the ratio of the stress drop to the applied stress at the moment of rupture becomes very small at that place, that is, at the lower crust and the upper mantle.

(*3*) Near the earth's surface, at the depth of 0-20 km, considerable part of the initial stress will be released by one major shock. Since the overspreading layers on the fracture zone are constituted with rather ductile materials and the strength of the layers is extremely low, the 'remained stress' at the fracture will be relaxed gradually. This process of stress-relaxation will cause the aftershock sequence as was suggested by the experimental study of the fracture of rocks in the preceding section.

(*4*) Since the strength of material of the earth's interior increases rapidly with depth and the fracturing becomes difficult to occur, the role of stress concentration seems to become very important for the generation of the fracture in the seismic region deeper than 20 km. In such depth, the most part of the initial stress can not be released by one major rupture, then the stress state of the ruptured region will only slightly be changed after the major rupture. Another state of high stress concentration will soon be reproduced. After the moderate period from the first major shock, the next major shock will follow. This will be the mechanism of the occurrence of earthquake-cluster type which is predominant at the depths of 20-80 km of seismic region.

(5) In the region deeper than 80 km, the shear faulting becomes quite difficult to occur by the effect of the increased friction (Orowan [1960]). The high stress concentration or large stress accumulation will also be impossible, because the materials become ductile and the strength is much lowered. Moreover, the broken surface will soon be healed by the effects of high pressure and high temperature. Considering above circumstances, it seems quite difficult that one major shock follows soon after another shock. Then isolated-shock is predominant in this deep region.

(6) Put the stress drop at rupture $\delta p_n = p_n - p_{n-1}$ and the elastic constant $\mu$, where $p_n$ and $p_{n-1}$ are the initial and 'remained stress' respectively, the energy release $E_n$ is given as follows,

$$E_n = \int_{p_{n-1}}^{p_n} p \frac{dp}{\mu} = \frac{(p_n + p_{n-1})}{2} \frac{\delta p_n}{\mu} \approx p_n \frac{\delta p_n}{\mu} .$$

Even if the absolute value of the stress drop $\delta p_n$ are same for every shock, the quantity of the energy release are quite different from one another according to the initial stress $p_n$. Therefore it must be noted neather the initial stress nor the strain-release can be estimated from the quantity of energy release of the earthquake.

(7) From the viewpoint of the fracture theory of rocks, it is rather relative sense that an earthquake is shallow focus shock or deep focus one. If a group of minor shocks is generated at the relatively shallow seismic region, say at the depth of 10-20 km, where the major shock will be accompanied with after-shock sequence, it is expected that this group probablly show the nature of the deep focus earthquake group, that is, type of earthquake cluster. Of course, it must be noted that the variation of the strength and the visco-elastic property with depth is the absolute nature of the seismic region, but not relative one.

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